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## Tarefa 8:

1)

O modelo probabilístico adotado foi o Binomial, visto que a experiência é repetida um número fixo ( $n$ ) de vezes, sempre nas mesmas condições, de tal forma que as probabilidades  $p$  e  $q$  ( $1-p$ ) se mantêm inalteradas a cada repetição. ~ slide 6 (Aula 4)

$$f(x, p) = \Pr(X=x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x},$$

para  $x = 0, 1, 2, \dots, n$

$$f(x, \theta) = \binom{n}{x} \cdot \theta^x \cdot (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^n f(x_i, \theta) = f(x_1, \theta) \cdot f(x_2, \theta) \cdot \dots \cdot f(x_n, \theta)$$

$$= \prod_{i=1}^n \binom{n}{x_i} \cdot \theta^{x_i} \cdot (1-\theta)^{n-x_i}$$

$$L(\theta) = \binom{n}{x_1} \cdot \theta^{x_1} \cdot (1-\theta)^{n-x_1} \cdot \binom{n}{x_2} \cdot \theta^{x_2} \cdot (1-\theta)^{n-x_2} \cdot \dots \cdot \binom{n}{x_n} \cdot \theta^{x_n} \cdot (1-\theta)^{n-x_n}$$

$$L(\theta) = \prod_{i=1}^n \binom{n}{x_i} \cdot \theta^{x_i} \cdot (1-\theta)^{n-x_i} //$$

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$$\ln(L(\theta)) = \ln\left(\pi\binom{n}{x_i}\right) + \sum x_i \cdot \ln(\theta) + \sum(n-x_i) \cdot \ln(1-\theta),$$

$$\frac{d(\ln(L(\theta)))}{d\theta} = \cancel{\phi} + \sum x_i \cdot \frac{1}{\theta} + \sum(n-x_i) \cdot \frac{(-1)}{1-\theta},$$

$$\frac{d(\ln(L(\theta)))}{d\theta} = \cancel{\phi}$$

$$\frac{\sum x_i}{\theta} - \frac{\sum(n-x_i)}{1-\theta} = \cancel{\phi}$$

$$\frac{(1-\theta) \cdot \sum x_i}{\theta \cdot (1-\theta)} - \frac{(\theta) \cdot \sum(n-x_i)}{\theta \cdot (1-\theta)} = \cancel{\phi}$$

$$(1-\theta) \cdot \sum x_i - \theta \cdot \sum(n-x_i) = \cancel{\phi}$$

$$\sum x_i - \theta \cdot \sum x_i - \theta \cdot \sum(n-x_i) = \cancel{\phi}$$

$$-\theta \cdot \sum x_i - \theta \cdot \sum(n-x_i) = -\sum x_i$$

$$\theta \cdot (\sum x_i + \sum(n-x_i)) = \sum x_i$$

$$\theta = \frac{\sum x_i}{\sum x_i + \sum(n-x_i)}$$

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## 2) Modelo Probabilístico: Distribuição de Bernoulli

$$f(x) = P(X=x) = p^x \cdot (1-p)^{1-x}, \text{ onde } x=0,1 //$$

$$f(x, \theta) = \theta^x \cdot (1-\theta)^{1-x} //$$

$$L(\theta) = f(x_1, \theta) \cdot f(x_2, \theta) \cdot \dots \cdot f(x_n, \theta)$$

$$= \prod_{i=1}^n f(x_i, \theta) //$$

$$L(\theta) = [\theta^{x_1} \cdot (1-\theta)^{1-x_1}] \cdot [\theta^{x_2} \cdot (1-\theta)^{1-x_2}] \cdot \dots \cdot [\theta^{x_n} \cdot (1-\theta)^{1-x_n}] //$$

$$L(\theta) = \theta^{\sum x_i} \cdot (1-\theta)^{\sum (1-x_i)} //$$

$$\frac{d(\ln(L(\theta)))}{d\theta} = \emptyset$$

$$\ln(L(\theta)) = \ln(\theta^{\sum x_i} \cdot (1-\theta)^{\sum (1-x_i)})$$

$$= \ln(\theta^{\sum x_i}) + \ln((1-\theta)^{\sum (1-x_i)})$$

$$= \sum x_i \cdot \ln(\theta) + \sum (1-x_i) \cdot \ln(1-\theta) //$$

Derivando e igualando a zero:

$$\sum x_i \cdot \frac{1}{\theta} + \sum (1-x_i) \cdot \frac{-1}{1-\theta} = \emptyset$$

$$\frac{\sum x_i}{\theta} - \frac{\sum (1-x_i)}{1-\theta} = \emptyset$$

$$(1-\theta) \cdot \sum x_i - \theta \cdot \sum (1-x_i) = \emptyset$$

$$\sum x_i - \theta \cdot \sum x_i - \theta \cdot \sum (1-x_i) = \emptyset$$

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$$+ \theta \cdot (\sum x_i + \sum (1-x_i)) = + \sum x_i$$
$$\theta = \frac{\sum x_i}{\sum x_i + \sum (1-x_i)} \approx \frac{\sum x_i}{\sum x_i + \sum c} - \frac{\sum x_i}{c}$$

$c = \text{constante}$

### 3) Modelo Probabilístico: Distribuição Geométrica //

$$f(x) = \Pr(X=x) = (1-\theta)^{x-1} \cdot \theta, \text{ onde } x=1, 2, 3, \dots$$

$$f(x, \theta) = (1-\theta)^{x-1} \cdot \theta //$$

$$L(\theta) = \prod_{i=1}^n f(x_i, \theta) //$$

$$L(\theta) = (1-\theta)^{\sum (x_i-1)} \cdot n \cdot \theta //$$

$$\frac{d(\ln(L(\theta)))}{d\theta} = \phi$$

$$\begin{aligned} \ln(L(\theta)) &= \ln((1-\theta)^{\sum (x_i-1)}) + n \cdot \ln(\theta) \\ &= \sum (x_i-1) \cdot \ln(1-\theta) + n \cdot \ln(\theta) // \end{aligned}$$

Derivando e igualando a zero:

$$\frac{\sum (x_i-1) \cdot (-1)}{1-\theta} + \frac{n}{\theta} = \phi$$

$$+ \sum (x_i-1) = + \frac{n}{\theta}$$

$$\theta \cdot \sum (x_i-1) = (1-\theta) \cdot n$$

$$\theta \cdot (\sum (x_i-1) + n) = n$$

$$\theta = \frac{n}{\sum (x_i-1) + n} //$$

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4) O modelo probabilístico adotado foi a Distribuição de Poisson, frequentemente utilizada na modelagem de eventos raros.

$$f(x) = \text{Pr}(X=x) = \frac{\lambda^x \cdot e^{-\lambda}}{x!}, \text{ onde } x=0,1,2,\dots$$

$$f(x, \theta) = \frac{\theta^x \cdot e^{-\theta}}{x!} //$$

$$L(\theta) = \frac{\theta^{\sum x_i} \cdot e^{-n\theta}}{\prod_{i=1}^n x_i!} //$$

$$\frac{d(\ln(L(\theta)))}{d\theta} = \emptyset$$

$$\begin{aligned} \ln \left( \frac{\theta^{\sum x_i} \cdot e^{-n\theta}}{\prod_{i=1}^n x_i!} \right) &= \ln(\theta^{\sum x_i} \cdot e^{-n\theta}) - \ln(\prod_{i=1}^n x_i!) = \\ &= \sum x_i \cdot \ln(\theta) + (-n\theta) \cdot \ln(e) - \ln(\prod_{i=1}^n x_i!) \\ &= \sum x_i \cdot \ln(\theta) - n\theta - \ln(\prod_{i=1}^n x_i!) // \end{aligned}$$

Derivando e igualando a zero:

$$\frac{\sum x_i - n}{\theta} = \emptyset$$

$$\frac{\sum x_i}{\theta} = n$$

$$\theta = \frac{\sum x_i}{n}$$

$$\theta = \frac{\sum x_i}{n} //$$